FRACTAL MODEL OF FOREST FIRE SPREAD IN THE INITIAL STAGE

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ABSTRACT:

In this article, fractal theory is applied to further study the spread of forest fires. The fractal relations under variable wind velocities, wind directions, and level and isotropic fuel beds are analysed. A fractal model is presented on the basis of self-similarity of fractal curve and the feature of the uneven boundary of fire field. A area formula of fire field is dedused from 1. 2618 coch curve. It turns out that average errors of relative values are small.the wider a fire field is, the smaller a relative error will be in calculation. The model is sufficiently simple, quite flexible, rather accurate and convenient to deal with forest fire spread in the initial stage.

KEY WORDS:

Boundary of fire field; fractal model; fractal operation;area of forest fire spread.

INTRODUCTION:

Forest fire spread is one of the important contents of forest fire behaviour. A lot of research work has been done on it.Especially now,more and more attentions are paid to it and the research on it is becoming deeper and deeper.In 1960's,the rate of forest fire spread was:$R=Ro.Kw.Ks/\cos\Phi$ .In the early 1970's,using the principle of the conservation of energy, Fransden derived a rate of spread equation for an infinitely long, flat fire front. By combining Fransden's ideas with experimental data from wind tunnel fires and further theory, Rothemel derived equations for the rate of spread and reaction intensity for a given constant fuel type,topographical and weather conditions for a flat fire front.$R=[I_R\xi\ (1+\Phi_w+\Phi_s)]/\rho_{b}\varepsilon\ Qig.$[2]In the late 1970's and early 1980's, with the rapid development of science and technology,computors were used to study numerical simulation of forest fires. For example, G. D Richards had done some work on it. The model he gave was based on nth order reaction theory and it dealt with variations in wind velocity, forest properties, fire breaks and other phenomena of interest to the fire controller.[3]

The development of physical theory has greatly stimulated the study on forest fire spread. In recent years,percolation theory has been applied to a wide range of research on it. Staffer.D.'s model was based on this theory.[4]He simulated the spread of forest fire with computors; Mackay.G and Jan. N, with the same theory approached forest fires as critical phenomena. Von Niessen.W and Blumen discussed the dynamic simulation of forest fires.[5]

Up to now, the analytical approximation most often used is that of an ellipse.[6] Other approximations that have been used include double ellipses[7]and parabola.[8]Variations from these shapes in actual fires are mainly attributed to: (1) the uneven boundary of fire field in the initial stage of forest fire spread. (2)when a forest fire becomes big, there are such phenomena as spotting and fire explosion. As a result, calculating the areas of fire fields with the above models, average errors are usually big.

Fractal theory is a useful, new method of solving complicated problems. In the past few years, it's been widely accepted and with it, people have solved a series of knotty questions in many fields. In this article, fractal theory is applied to further research forest fire spread. A fractal model is set up here based on the self-similarity of fractal curve and the feature of uneven boundary of fire field. A area formula is given to calculate the area of forest fire spread in the initial stage. Its advantages are shown through the calculations of some actual fire fields.

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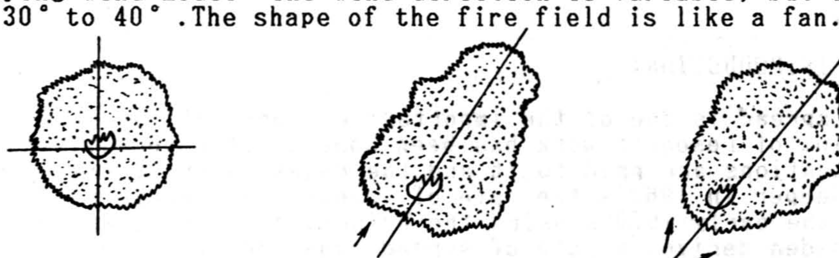
As we know, the spread of forest fires has much to do with fuel, wind velocity and direction, terrain, topography and moisture. As to the fuels that affect the spread of forest fires, there are many complex factors: (1) size of fuel; (2) density of fuel; (3) fuel type; (4) fuel moisture content. Under the homogeneous fuel(the above four indexes are essentially the same) the spread of forest fires can be induced into three normal models:[9]

1.Quiet wind model:When no wind or wind velocity is very small, the fire propagates as an expanding circle from the fire starting spot. The circle boundary is uneven.[Fig.1]a

2. Powerful wind model: Wind direction is unchanged, wind velocity is powerful, the shape of the fire field is ovoid or of willow leaf[Fig.1]b

3. Swaying wind model: The wind direction is variable, but it sways between 30° to 40° .The shape of the fire field is like a fan.[Fig.1] c



a.quiet wind model b.powerful wind model c.swaying wind model

Fig.1 Forest fire spread in the initial stage

Fig.1 obviously shows that fire fields consist of fire front, fire tail, and fire flank except the quiet wind model, and that the three boundaries of fire fields are uneven. These uneven curves have the characteristics of continuity and undifferentiation. So are the fractal curves in the fractal theory. Thus, it's more identical to substitute fractal curves for the boundaries of fire fields than the smooth curves used in the past. That's to say, fractal curves are more similar to the boundary of fire fields. The fractal model to be mentioned in the following is founded by the principle of self-similarity of fractal theory and fractal coch snowflake curve.[10, 11, 12]

As to the quiet wind model, the point of ignition is centred, average radius as radius, draw a circle. An equilateral triangle is made in the circle, the triangle as the fountainhead polygon, draw a coch curve which replaces the quiet wind model.[Fig.2] Fractal dimension of coch curve is 1.2618. The area is 8/5 times of the fountainhead polygon.

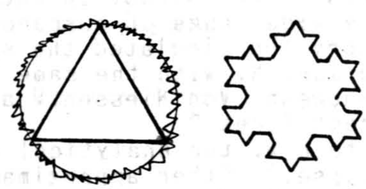


Fig.2

As to the powerful wind and swaying wind model, rhombus as fountainhead polygon, the distance from fire frant to fire tail as long diagonal line. the distance between the two fire wings as short diagonal line. Fractal dimension can be 1. 2618 or other dimensions such as 1.2091. See[Fig.3 a]

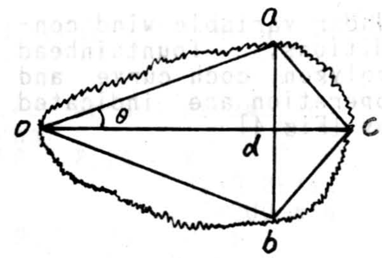


Fig.3 a

The area of the fountainhead polygon is:

$$A_o=k(ab)^2/2 \quad (k=oc/ab) \tag{1}$$

Operate by 1.2618 fractal curve:
First, to each side of the rhombus add an equilateral triangle each side of which is equal to 1/3 of the side of the rhombus, we get the polygon illustrated in [Fig.3 b]

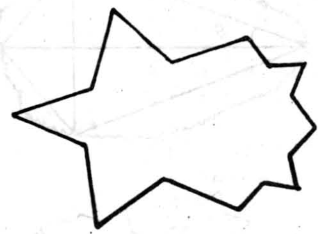


Fig.3 b

$oa=ab/2Sin\theta$; $ac=ab(1/4Sin^2\theta + k^2 - kcty\theta)^{\frac{1}{2}}$

The area of the polygon after the first operation

$$A_1=A_o[1+(2\sqrt{3}/18)(1/2Sin^2\theta\ k+k-cty\theta)] \tag{2}$$

Second, to each side of the above polygon add another equilateral triaggle each side of which is equal to 1/3 of the side of the polygon, we get the polygon illustrated in [Fig.3 c]
The area of the polygon after the second operation

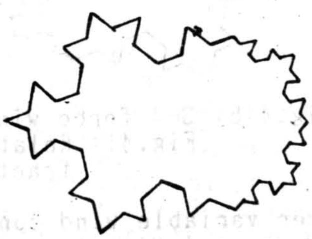


Fig.3 c

$$A_2=A_o[1+(1/2Sin^2\theta\ k+k-cty\theta)(4/9+16/81)] \tag{3}$$

Go on with the same operation until n time.

$$A_n=A_o[1+(\sqrt{3}/5)(1/2Sin^2\theta\ k+k-cty\theta)] \tag{4}$$

$$K_3=1+(\sqrt{3}/5)(1/2Sin^2\theta\ k+k-cty\theta) \tag{5}$$

When the shape and boundary of fire field change, the fractal dimension of the fractal curve should be changed relatively so that the fractal curve may better fit the fire field boundary.e.g.if the wind power is 7 or over 7, it's better to use 1.2091 fractal dimension. When the fractal dimension is equal to 1.2091, with the same operation, we get the formula.

$$A_n=A_o[1+(7\sqrt{3}/18)(1/2Sin^2\theta\ k+k-cty\theta)] \tag{6}$$

$$K_3=1+(7\sqrt{3}/18)(1/2Sin^2\theta\ k+k-cty\theta) \tag{7}$$

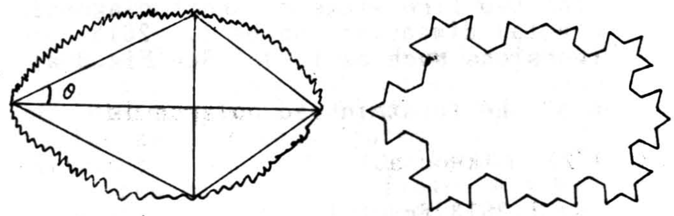
In formulas (4) and (6) , An has much to do with θ ,ab and oc;ab and oc have much to do with R_f. An is closely related to the choosing of fountainhead polygon. θ in the fountainhead polygon depends on wind power. Through a large quatity of arranged and related data, Table 1 is given showing the relations between the wind power and the fountainhead polygon.

Table 1:

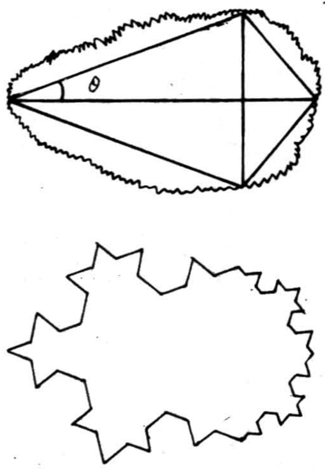
wind force	θ	R_f a)	$R_b=K_1R_f$	$R_1=K_2R_f$	K=oc/ab
1--2	25°	R_f	$0.05R_f$	$0.47R_f$	10/9
3--4	20°	R_f	$0.04R_f$	$0.36R_f$	10/7
5--6	15°	R_f	$0.03R_f$	$0.27R_f$	10/4
7-over 7	10°	R_f	$0.02R_f$	$0.18R_f$	10/3

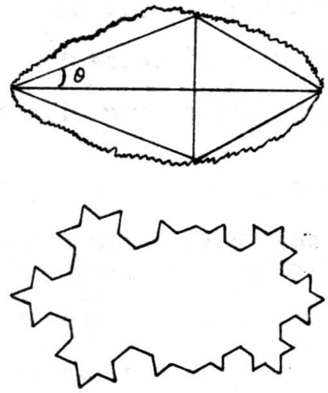
a) $R_f=R_o(k_w.K_s)/cos\theta$

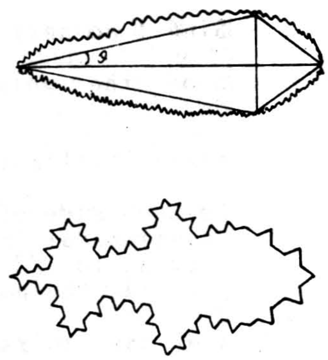
Under variable wind conditions, fountainhead polygon, coch curve and operation are indicated by [Fig.4]



[Fig.4] a: 1-2 force wind







[Fig.4]b: 3-4 force wind c: 5-6 force wind d: 7-over 7 force wind

[Fig.4]: Relations among fountainhead polygon, fractal dimension and operation

Under variable wind condi tions,the selections of fountainhead polygons and fractal dimentions,and areas after operations are shown in Table 2.

Table 2:

wind force	K	fountainhead polygon	A_0	D_f	A_n	K_3
0	1.0	equilateral triangle	$3\ 3(ab)^2/16$	1.2618	1.60A	1.60
1-2	1.1	rhombus	$(ab)^2/2k$	1.2618	1.52A	1.52
3-4	1.4	rhombus	$(ab)^2/2k$	1.2618	1.58A	1.58
5-6	2.5	rhombus	$(ab)^2/2k$	1.2618	1.60A	1.60
7-over 7	3.3	rhombus	$(ab)^2/2k$	1.2091	1.58A	1.58

We can see from Table 2 that:When θ is over 10°, $A_n=K_3A_0$.In the light of fractal, the self-similarity in unscaling district can be considered as the same. The area of coch snowflake of the fountainhead polygon is its K_3 times, which is approximate to the area of the fire field. K_3 depends on the wind power and the chosen fractal dimension. Thus, the area formula of the spread of the forest fire field is obtained.

$\because \quad R_b=k_1R_f;\ R_1=k_2R_f;\quad k=oc/ab=(1+k_1)/2k_2.$

$\therefore \quad A_0=k_2(1+k_1)R_f^2\triangle T^2 \qquad (8)$

$\therefore \quad A_n=k_3A_0=k_3k_2(1+k_1)R_f^2\triangle T^2 \qquad (9)$

COMPARISIONS AND DISCCUSIONS

In the Spring of 1982, the period of fire prevention, Hulunbeire Meng fire situation. Comparisions of fire recorded areas and the areas calculated by the fractal model. Table.3

order	place	time	fire type	recorded area(ha)	counted area(ha)	error	parameter
1	Wukeli gully	May.20 12:50-16;10	wild fire	30.00	32.50	8.2%	W_4 R_f=3.8m/min. $\triangle T$=200 min.
2	Datie 108km	May.20 14:10-14:45	situ-ation fire	0.38	0.39	3.8%	W_{1-2} R_f=2.1m/min. $\triangle T$=35 min.
3	Dong-hetao	Jun.1 12:00-14:00	wild fire	7.47	11.97	59.4%	W_{2-3} R_f=3.8m/min. $\triangle T$=120 min.
4	Dayin-shuji he	Jun.1 12:00-20:00	fire	666.70	627.18	5.9%	W_{2-3} R_f=6.5m/min. $\triangle T$=480 min.
5	Ailin-138 gully	May.20 9:37-15:04	fire	25.00	23.28	6.9%	W_4 R_f=1.9m/min. $\triangle T$=327 min.
6	Yimin-xipo	Apr.24 13:40-22:30	prai-rie fire	1.20 $\times10^4$	1.16 $\times10^4$	3.2%	W_4 R_f=27.1m/min. $\triangle T$=530 min.
7	Yimin-xipo	Apr.25 11:40-6:00	prai-rie fire	5.92 $\times10^4$	6.05 $\times10^4$	2.2%	W_{7-8} R_f=43.5m/min. $\triangle T$=1100 min.

In Table.3,the errors of the prairie great fires are very small for the fuel distributing situations on the prairie are very much like the above given fuel distributing situations.On the other hand,the areas of these two prairie fires are very big, the boundary tortuosity of the fire field of large area coincides with that of fractal curve.That's to say, when the area of a fire field is big, its unscaling district is big, too. Its boundary is more similar to the chosen fractal curves. So the error is very small. It conforms to the self-similarity of fractal theory. Not including the biggest and the smallest errors, the average relative error is only 5. 6%. These records were taken down by some experts in fire prevetion with their eyes or in the air, it's hard to avoid any mistakes. Thus the comparisions are to be considered as reference only.

Table.4: Comparisions of calculating values with different models.

order	record (ha)	fractal count	fractal error	ellipse count	ellipse error	dou.ellipse count	dou.ellipse error	parabola count	parabola error
1	30.00	32.50	8.2%	129.63	332.0%	45.54	51.8%	24.89	17.0%
2	0.38	0.39	3.8%	1.59	320.0%	0.51	35.7%	0.35	7.0%
3	7.47	11.91	59.4%	61.51	724.0%	16.40	119.7%	9.98	21.0%
4	666.70	627.18	5.9%	2184.64	227.6%	767.75	15.0%	470.08	29.0%
5	25.00	23.28	6.9%	90.30	261.0%	31.73	27.0%	19.39	22.0%
6	1.20 $\times10^4$	1.16 $\times10^4$	3.2%	4.63 $\times10^4$	286.0%	1.63 $\times10^4$	35.6%	1.20 $\times10^4$	0.0%
7	5.92 $\times10^4$	6.05 $\times10^4$	2.2%	2.51 $\times10^5$	323.0%	1.29 $\times10^5$	61.7%	5.04 $\times10^4$	15.0%
mean error			5.6%		297.7%		42.2%		16.4%

$$A_{ellipse}=6K_2(1+K_1)R_f^2\triangle T^2 \quad (10)$$

$$A_{double\ ellipse}=(\pi/8)[(1+K_2)^2+(K_2+K_1)^2]R_f^2\triangle T^2 \quad (11)$$

$$A_{parabola}=(4/3)(1+k_1)k_2R_f^2\triangle T^2 ty\theta \quad (12)$$

Fractal model takes triangle and rhombus as fountainhead polygon. The area of fire field is replaced by closed surface composed of coch curves (D_{r1}=1.2618; D_{r2}=1. 2091) .The area is decided by R_r, $\triangle T$ and θ . Therefore, R_r, $\triangle T$ and wind power require accurate values on the spot. When the area of fire field is calculated by fractal curves, K_3 is an important factor that affects the error value. K_3 is depended on the suitably chosen fountainhead polygon and the fractal dimension according to variant wind. So it's convenient in practice.

CONCLUSIONS

So far as I know,the approximate calculations of forest fire spread are generally indicated by ellipse, double ellipse, tear-drops, ovoids and parabola,the characteristic of which is to use smooth curves to replace the uneven boundary of fire field. But fractal curves are selden used. Through the work I've done on it, I find out the advantages of the fractal model.Here we may draw the conclusions:

A. using fractal curves to replace the uneven boundary of a fire field is more actual than using smooth curves, for the uneven boundary indicates in a manner more similar to the defined fractal curves.

B. Different fountainhead polygons can be selected by different shapes of fire fields; and different fractal dimensions of fractal curves can be chosen by different boundaries of fire fields in operation. So this model can be used conveniently and widely.

C. Counting results are more accurate,and errors are smaller.The larger a area of a fire field is ,the smaller its error will be.

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