MATHEMATICAL MODEL OF SEVERE WILDLAND FIRES

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ABSTRACT

Severe wildland fires are modeled based on the fundamental theories of combustion and fluid mechanics. The k-ε model of turbulence is employed. A model of severe wildland fuels is given by a trade-off between computational demands and a simple but realistic model. The field is divided into inner region around the burning ring and outer region. And then the problem is analysed by using the method of matched asymptotic expansions. The outer problem is reduced to a direct problem, and the inner problem is reduced to a quasi-2D problem of flow, heat transfer and combustion.

KEYWORDS: Wildland fire Combustion Heat transfer Perturbation methods.

INTRODUCTION

The severe wildland fire behavior has highlighted the need for a method of assessing the danger of extremely severe fires and predicting their behavior. At the moment the data that is available for prototype burns and laboratory scaling is too scant to provide the necessary correlation and there does not seem much hope that it will be forthcoming in the foreseeable future. The numerical simulation is seen as the cheapest and yet realistic approach to understanding severe fire behaviour.[1,2,3] For this reason, a mathematical model is needed that will predict the growth and spread of large fires.

ANALYSIS BY THE METHOD OF MATCHED ASYMPTOTIC EXPANSIONS

Wildland fires generally spread in the form of a narrow burning ring

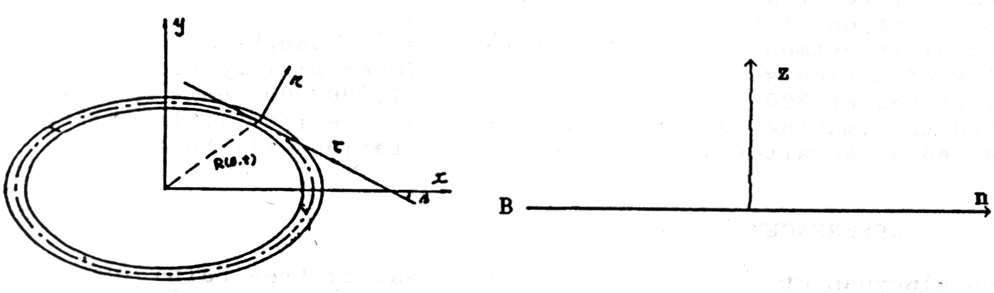

Fig.1 A fire spreads in a narrow burning ring enlarging with time (1a) and the cross plane (1b).

enlarging with time, with the ring curvature radius $R(\theta,t)\gg 1$ being much larger than its cross-scale of $O(1)$, as shown in fig.1 . A Cartesian coordinate system is defined, with x-axis along the wind direction, z-axis in the vertical direction. To solve the problem by the method of matched asymptotic expansions, the flow field is divided into two separate asymptotic regions, i.e. the inner region of slender ring around the burning ring with the cross-scale of $O(1)$ and the outer region.

A. Outer problem

In the outer region, $x,y,z=O(R)$. According to the conservation of mass and energy, the variations of velocity and temperature due to the fire are of $O(1/R)$. The asymptotic expansions of outer velocity and temperature are

$$\vec{v}=U_w(z)\vec{i} + \frac{1}{R}\vec{v}_1(x,y,z,t)+o(\frac{1}{R}) \qquad T=T_\infty+\frac{1}{R}T_1(x,y,z,t)+o(\frac{1}{R}), \tag{2-1}$$

where $U_w(z)$ is the wind speed. The energy equation reduces to

$$\frac{\partial}{\partial t}T_1+U(z)\frac{\partial}{\partial x}T_1=\frac{1}{R_e\ P_r}\Delta T_1, \tag{2-2}$$

where R_e and P_r are Reynolds number and Prandtl number of outer problem. Noticing that the Reynolds number $R_e=\frac{UR}{\nu}=o(R)$, one can see that $T_1=0$. The governing equations of $\vec{v}_1(x,y,z,t)$ are

$$\nabla\ \vec{v}_1=0 \qquad \frac{\partial}{\partial t}\vec{\omega}_1+U(z)\frac{\partial}{\partial x}\vec{\omega}_1=\frac{1}{R_e}\Delta\vec{\omega}_1 \qquad \vec{\omega}_1=\Delta x\vec{v}_1, \tag{2-3}$$

so that $\vec{\omega}_1=0$.

Intuitively, the disturbance of the narrow burning ring in the outer region acts to repel of the fluid. In addition, the ring shrinks to the line Γ: $r=R(\theta,t)$, the center line of the ring, as seen by an outer observer. Therefore, it is inferred that $\vec{v}_1$ can be represented by a concentrated line source along the line Γ, i.e.

$$\vec{v}_1=\nabla\varphi \qquad \varphi=\frac{1}{\pi}\int_\Gamma \frac{\sigma(s_1,t)ds_1}{\sqrt{(x_1-x)^2+(y_1-y)^2+z^2}}, \tag{2-4}$$

From the conservation of mass, it can be inferred that the unknown line source $\sigma(s_1,t)$ is the distribution of the mass expansion of the inner region.[4]

B. Inner problem of gaseous part

To account for the interactions between the fuels and the gas in the inner region, the fuels and the gas are considered as the solid and the gaseous part of a porous bed respectively. A local Cartesian coordinate system (n,τ,z) is set also with its origin at $P(s)$ on Γ and with the n- and τ-axes normal and tangent to Γ at P(s) respectively in the horizontal plane, as sketched in fig. 1. The asymptotic expansions for density ρ, mass fractions of fuel and oxygen m_{fu}, m_{ox}; velocity components u, v, w in the coordinate system (n,τ,z); enthalpy h, turbulence energy k, and turbulence energy dissipation ε can be written in the same form

$$\phi=\phi^{0}(n,\tau,z,t)+\frac{1}{R}\phi^{1}(n,\tau,z,t)+o(1/R). \quad (2\text{-}5)$$

Noticing the fact that the scales of the inner region are O(1) in the n- and z-directions, and O(R) in the τ-direction, the governing equations to first order approximation for the gas phase in the inner region are as follows

continuity equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial n}+\frac{\partial(\rho w)}{\partial z}=\{\dot{S}\} \quad (2\text{-}6a)$$

momentum equations

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial n}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial P}{\partial n}+2\frac{\partial}{\partial n}(\mu_t\frac{\partial u}{\partial n})+\frac{\partial}{\partial z}[\mu_t(\frac{\partial w}{\partial n}+\frac{\partial u}{\partial z})]+\{u\dot{S}\} \quad (2\text{-}6b)$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho vu)}{\partial n}+\frac{\partial(\rho vw)}{\partial z}=\frac{\partial}{\partial n}(\mu_t\frac{\partial v}{\partial n})+\frac{\partial}{\partial z}(\mu_t\frac{\partial v}{\partial z})+\{v\dot{S}\} \quad (2\text{-}6c)$$

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho wu)}{\partial n}+\frac{\partial(\rho ww)}{\partial z}=-\frac{\partial P}{\partial z}+\frac{\partial}{\partial n}[\mu_t(\frac{\partial w}{\partial n}+\frac{\partial u}{\partial z})]+2\frac{\partial}{\partial z}(\mu_t\frac{\partial w}{\partial z})-\rho g_r+\{w\dot{S}\} \quad (2\text{-}6d)$$

energy equation

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial(\rho uh)}{\partial n}+\frac{\partial(\rho wh)}{\partial z}=\frac{\partial}{\partial n}(\Gamma_h\frac{\partial h}{\partial n})+\frac{\partial}{\partial z}(\Gamma_h\frac{\partial h}{\partial z})+Q_r+\{h_{in}\dot{S}-Q_t\} \quad (2\text{-}6e)$$

species equations

$$\frac{\partial(\rho m_{fu})}{\partial t}+\frac{\partial(\rho u m_{fu})}{\partial n}+\frac{\partial(\rho v m_{fu})}{\partial z}=\frac{\partial}{\partial n}(\Gamma_{fu}\frac{\partial m_{fu}}{\partial n})+\frac{\partial}{\partial z}(\Gamma_{fu}\frac{\partial m_{fu}}{\partial z})-\dot{R}_{fu}+\{\alpha_{fu}\dot{S}\} \quad (2\text{-}6f)$$

$$\frac{\partial(\rho m_{ox})}{\partial t}+\frac{\partial(\rho u m_{ox})}{\partial n}+\frac{\partial(\rho v m_{ox})}{\partial z}=\frac{\partial}{\partial n}(\Gamma_{ox}\frac{\partial m_{ox}}{\partial n})+\frac{\partial}{\partial z}(\Gamma_{ox}\frac{\partial m_{ox}}{\partial z})-\dot{R}_{ox} \quad (2\text{-}6g)$$

k-ε model of turbulence

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho uk)}{\partial n}+\frac{\partial(\rho wk)}{\partial z}=\frac{\partial}{\partial n}(\Gamma_k\frac{\partial k}{\partial n})+\frac{\partial}{\partial z}(\Gamma_k\frac{\partial k}{\partial z})+G-\rho\varepsilon \quad (2\text{-}6h)$$

$$\frac{\partial(\rho\varepsilon)}{\partial t}+\frac{\partial(\rho u\varepsilon)}{\partial n}+\frac{\partial(\rho w\varepsilon)}{\partial z}=\frac{\partial}{\partial n}(\Gamma_\varepsilon\frac{\partial\varepsilon}{\partial n})+\frac{\partial}{\partial z}(\Gamma_\varepsilon\frac{\partial\varepsilon}{\partial z})+\frac{\varepsilon}{k}(C_1G-C_2\rho\varepsilon), \quad (2\text{-}6i)$$

Flux model of heat radiation

$$\frac{\partial}{\partial n}(\frac{1}{a+s}\frac{\partial}{\partial n}q_n)=a(q_n-4E)+\frac{S}{2}(q_n-q_z) \quad (2\text{-}6j)$$

$$\frac{\partial}{\partial z}(\frac{1}{a+s}\frac{\partial}{\partial z}q_z)=a(q_z-4E)+\frac{S}{2}(q_z-q_n) \quad (2\text{-}6k)$$

$$Q_r = 2a(q_n + q_z - 2E) \qquad E = \sigma T^4, \tag{2-61}$$

with $\mu_t = c\rho k^2/\varepsilon$

$$G = \mu_t \{2[(\frac{\partial u}{\partial n})^2 + (\frac{\partial w}{\partial z})^2] + (\frac{\partial u}{\partial n} + \frac{\partial w}{\partial z})^2 + (\frac{\partial v}{\partial n})^2 + (\frac{\partial v}{\partial z})^2 - \frac{\beta^g{}_r}{T\sigma_h}\frac{\partial T}{\partial z}\}$$

$$c = 0.09 \qquad c_1 = 1.44 \qquad c_2 = 1.92 \qquad \Gamma_h = \Gamma_{fu} = \Gamma_{ox} = \Gamma_k = \Gamma_\varepsilon = 1, \tag{2-7}$$

where $\dot{S}$ stands for the mass flow rate from the solid to the gas, the source terms in brackets, { }, are valid only within the porous bed; Q_t is the heat transfer rate from gas to solid; h_{in} refers to the enthalpy of the released gas

$$h_{in} = \dot{S}(\alpha_{fu} H_{fu} + \alpha_{H2O} H_{H2O} + C_p T_{fu}), \tag{2-8}$$

α_{fu} and α_{H2O} are mass fractions of fuel and water vapor in the released gas; σ is Stefan-Boltzmann's constant, a, S are absorption and emission coefficients of medium in unit length. For the sake of brevity, the superscript of the first-order solution is omitted in the above equations.

These governing equations can be written into a general form

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho u\phi)}{\partial n} + \frac{\partial(\rho w\phi)}{\partial z} = \frac{\partial}{\partial n}\left[\Gamma_\phi \frac{\partial\phi}{\partial n}\right] + \frac{\partial}{\partial z}\left[\Gamma_\phi \frac{\partial\phi}{\partial z}\right] + S_\phi, \tag{2-9}$$

where ϕ stands for dependent variables, Γ_ϕ for the exchange coefficient and S_ϕ for the source term for the corresponding variable ϕ.

Weak boundary conditions are employed at all open boundaries, i.e.

as $r^2 = n^2 + z^2 \to \infty$,

for $v_N < 0$, $\quad u = U_W \cos\beta + \frac{\sigma(s,t)}{\pi r}\cos(\theta) \quad v = U_W \sin\beta + \frac{\sigma(s,t)}{\pi r}\cos(\theta) \quad w = 0 \quad h = h_\infty$

$$m_{fu} = m_{fu\infty} \qquad m_{ox} = m_{ox\infty} \qquad \frac{\partial k}{\partial N} = \frac{\partial \varepsilon}{\partial N} = \frac{\partial q_n}{\partial N} = \frac{\partial q_z}{\partial N} = 0;$$

for $v_N \geq 0$, $$\frac{\partial u}{\partial N} = \frac{\partial v}{\partial N} = \frac{\partial w}{\partial N} = \frac{\partial h}{\partial N} = \frac{\partial m_{fu}}{\partial N} = \frac{\partial m_{ox}}{\partial N} = \frac{\partial k}{\partial N} = \frac{\partial \varepsilon}{\partial N} = \frac{\partial q_n}{\partial N} = \frac{\partial q_z}{\partial N} = 0, \tag{2-10}$$

where N is the normal to the open boundary, β is the angle between τ- and x-axes, $\theta = tg^{-1}(z/n)$. The boundary conditions on the wall are given as z=0,

$$u = v = w = k = \varepsilon = 0 \qquad \frac{\partial h}{\partial z} = \frac{\partial m_{fu}}{\partial z} = \frac{\partial m_{ox}}{\partial z} = 0$$

$$\frac{\partial q_x}{\partial z} = 0 \qquad \frac{dq_z}{dz} = \frac{2\alpha}{2-\alpha}(q_z - E), \tag{2-11}$$

where α is the emission coefficient of the ground.

C. Model of wildland fuels

Wildland fuels are divided by kind, live degree, and spatial distribution. If all these factors are considered together, the fuel model must be too complicated to use. If only one factor is considered, the result will be deviated from the real procedure. Therefore, a compromise has to be made between accuracy and practicality. For this reason, the following assumptions are introduced:

(i) Wildland fuels can be roughly divided into two classes: (a) the fuel with its scale being of 1mm at least at one direction, such as grasses, leaves, etc., the time scale of its pyrolysis is small compared with that of fire behaviour; (b) the circular cylinder fuel with its diameter scale being 10mm. Both of the two class fuels consists of fuel and water with the mass fractions $\alpha_{fu}^{(i)}$, $\alpha_{H2O}^{(i)}$, i=1,2. For the scales of fuels are small compared with the cross-scale of the burning ring, these mass fractions are assumed as the continuous functions of space. For the later class, the pyrolysis rate and water vaporization rate with its temperature are given as follows from the chemical dynamic equations

$$\dot{S}_{fu}=\alpha_{fu}^{(2)}k_1\exp(-E_1/T) \qquad \dot{S}_{H2O}=\alpha_{H2O}^{(2)}k_2\exp(-E_2/T), \qquad (3\text{-}1)$$

where k_1, k_2, E_1, and E_2 are the reaction rate and energy parameters of fuels.[5]

(ii) The fuel vapor comes from vaporization of the pyrolysis of the porous fuel bed. A simple one-step irreversible reaction model is used to describe the combustion. A unit mass of fuel combines with s mass units of oxygen to give (1+s) units of products, s here refers to the stoichoimetric ratio of fuel and oxygen. As the mixing process is predominant in controlling the reaction rate for wildland fires, the local reaction rate is expressed with the Magnussen's version of Spalding's eddy-break-up model

$$R_{fu}=A\rho\frac{\varepsilon}{k}\min\left(m_{fu},\ \frac{m_{ox}}{s},\ \frac{m_{pr}}{B(1+s)}\right), \qquad (3\text{-}2)$$

where A and B are constants given the value 4 and 2 respectively.[6]

(iii) It is not possible to know the details of motion of gas within the porous bed in the small scale. Fortunately, our interest is on the relatively large scale motion from the practical point of view. The method of porosity is used to simulate the porous fuel bed, as proposed by Fan (1985, 1991).[7,8] The porosity is a proportion of a cell through which fluid can flow. Both the volume porosity and surface porosity are introduced. They express the proportions of volume and surface of a cell respectively, which are available for fluid flow in the cell. Porosity can be as factors incorporated into the discretized equations,[8]

$$\begin{cases}\dfrac{\partial(\rho\phi VP_{ov})}{\partial t}+\sum_i[(\rho\vec{v}\phi-\Gamma_\phi \mathrm{grad}\phi)\vec{A}P_{oi}]=(S_\phi VP_{ov})_P \\ i=e,s,w,n\end{cases} \qquad (3\text{-}3)$$

where V stands for the volume of a cell; Pov and Poi for the volume porosity and surface porosity of the cell respectively; the subscript i=e,s,w,n for the east, south, west, and north surfaces of the cell.

CONCLUSIONS

Sever wildland fires have been formulated by using the method of matched asymptotic expansions. The outer flow is reduced to a cool laminar flow to second-order approximation, consisting of the stratified uniform wind and the potential flow induced by a line source along the burning ring with its strength determined by the first-order inner problem. The inner region has been considered as a porous bed with fuels and gas. The inner problem of gaseous part is reduced as a quasi 2-D problem of flow, heat transfer and combustion in the inner region. The 3-D effect in the inner region is due to the interaction of mean flow and turbulent fluctuation.

In addition, the fire in inner region is different from the burning of material of uniform phase due to the existence of gas flow and combustion, and their interactions within the bed. The processes have been modeled by a set of governing equations of gas phase with appropriate modifications of their source terms and numerical treatment of introducing volumetric and surface porosity of each cell into the discretized equations.

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