F.R. Note No. 155/55

DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH AND FIRE OFFICES' COMMITTEE
JOINT FIRE RESEARCH ORGANIZATION

THE THERMAL INSULATION AFFORDED

BY A FIRE RETARDANT COATING

by

R. W. Pickard

Summary

An expression has been derived for the interface temperature rise of a combustible material with a protective layer when exposed to heat radiation. Analysis of the results gives a guide to the physical properties of an effective fire retardant paint and the conditions under which the expression gives valid results.

January, 1955.

Fire Research Station,
Station Road,
Boreham Wood,
Herts.

THE THERMAL INSULATION AFFORDED
BY A FIRE RETARDANT COATING

by

R. W. Pickard

1. Introduction

The amount of heat required to ignite a combustible surface can be increased by protecting it with a fire retardant paint layer. This provides thermal insulation and thus reduces the rate of rise of temperature of the combustible surface. This temperature rise depends not only on the time of exposure and intensity of radiation but also on the physical properties of the material and the protective layer. It is the purpose of this note to derive an expression for the temperature rise at the interface of a material and its protective layer and to discuss the factors upon which it depends.

A rigorous solution of the problem would be too complex to be useful for assessing the effectiveness of a fire retardant treatment and certain limiting assumptions have therefore been made:

(1) that the composite solid formed by the material and its protective layer is semi-infinite in extent;

(2) that the physical properties of the materials used are independent of temperature; and

(3) that there is no heat loss from the irradiated surface.

These assumptions will limit the validity of the results to short times of irradiation but the solution may however be used as a guide in determining the physical properties of a protective layer which is most likely to be effective in reducing the temperature rise of the protected surface.

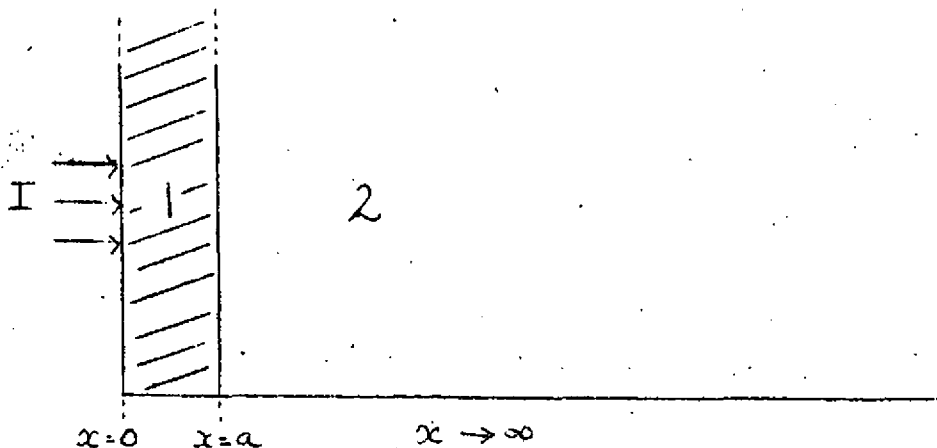## 2. Theoretical derivation of interface temperature rise

Fig. 1

In Fig. 1 material 2 represents the material to be protected and material 1 the protective layer which has a thickness a.

Let Θ_1, Θ_2 be the temperature rise at points x_1, x_2 in materials 1 and 2 respectively

Let K_1, K_2 be the thermal conductivities of materials 1 and 2

" k_1, k_2 " " " diffusivities " " 1 " 2

" I " " incident intensity of radiation

Then the equations to be satisfied are

$$\frac{\partial^2 \theta_1}{\partial x^2} = \frac{1}{k_1} \frac{\partial \theta_1}{\partial t} \qquad 0 \leq x \leq a \quad \dots\dots\dots\dots\dots(1)$$

$$\frac{\partial^2 \theta_2}{\partial x^2} = \frac{1}{k_2} \frac{\partial \theta_2}{\partial t} \qquad a \leq x \leq \infty \dots\dots\dots\dots\dots(2)$$

$$I = -K_1 \frac{\partial \theta_1}{\partial x} \qquad t > 0 \quad \dots\dots\dots\dots\dots(3)$$

Assuming no thermal resistance at the interface

$$K_1 \frac{\partial \theta_1}{\partial x} = K_2 \frac{\partial \theta_2}{\partial x} \qquad x = a \quad \dots\dots\dots\dots\dots(4)$$

$$\theta_1 = \theta_2 \qquad x = a \quad \dots\dots\dots\dots\dots(5)$$

Defining $\bar{\theta} = \int_0^\infty \theta e^{-pt} dt$ the Laplace transform of θ then the subsidiary equations become

$$\frac{d^2 \bar{\theta}_1}{dx^2} = \frac{p}{k_1} \bar{\theta}_1 \qquad \dots\dots\dots\dots\dots(L1)$$

$$\frac{d^2 \bar{\theta}_2}{dx^2} = \frac{p}{k_2} \bar{\theta}_2 \qquad \dots\dots\dots\dots\dots(L2)$$

$$\frac{I}{p} = -K \frac{d\bar{\theta}_1}{dx} \qquad \dots\dots\dots\dots\dots(L3)$$

$$K_1 \frac{d\bar{\theta}_1}{dx} = K_2 \frac{d\bar{\theta}_2}{dx} \qquad \dots\dots\dots\dots\dots(L4)$$

$$\bar{\theta}_1 = \bar{\theta}_2 \qquad \dots\dots\dots\dots\dots(L5)$$

A solution of (L1) is

$$\bar{\Theta}_1 = A_1 e^{+q_1 x} + B_1 e^{-q_1 x} \qquad \dots\dots\dots(6)$$

A solution of (L2) is

$$\bar{\Theta}_2 = A_2 e^{q_2 x} + B_2 e^{-q_2 x} \qquad \dots\dots\dots(7)$$

A_1, A_2, B_1 and B_2 are constants to be determined from the boundary conditions.

Since $\Theta_2 \to 0$ as $x \to \infty$, $A_2 = 0$

The value of the remaining constants, determined from the boundary conditions stated in (L3), (L4) and (L5), are as follows:

$$A_1 = \frac{I e^{-q_1 a}(1 - \alpha)}{2p(K_2 q_2 \cosh q_1 a + K_1 q_1 \sinh q_1 a)} \qquad \text{where } \alpha = \frac{K_2 q_2}{K_1 q_1}$$

$$B_1 = \frac{I e^{q_1 a}(1 + \alpha)}{2p(K_2 q_2 \cosh q_1 a + K_1 q_1 \sinh q_1 a)}$$

$$B_2 = \frac{I e^{q_2 a}}{p(K_2 q_2 \cosh q_1 a + K_1 q_1 \sinh q_1 a)}$$

Hence

$$\bar{\Theta}_1 = \frac{I[\cosh q_1(a-x) + \alpha \sinh q_1(a-x)]}{p[K_2 q_2 \cosh q_1 a + K_1 q_1 \sinh q_1 a]} \qquad \dots\dots\dots(8)$$

$$\bar{\Theta}_2 = \frac{I e^{q_2(a-x)}}{p[K_2 q_2 \cosh q_1 a + K_1 q_1 \sinh q_1 a]} \qquad \dots\dots\dots(9)$$

The Laplace transform of the interface temperature rise $\bar{\Theta}_i$ may be found by putting $x = a$ in equations (8) or (9), whence

$$\bar{\Theta}_i = \frac{I}{p(K_2 q_2 \cosh q_1 a + K_1 q_1 \sinh q_1 a)} \qquad \dots\dots\dots(10)$$

The inverse of (10) may be found by rewriting the equation in the exponential form whence

$$\bar{\Theta}_i = \frac{2I}{p\sqrt{p}\left(\frac{K_2}{\sqrt{k_2}} + \frac{K_1}{\sqrt{k_1}}\right)} \frac{e^{-q_1 a}}{1 + \gamma e^{-2q_1 a}} \quad \text{where} \quad \gamma = \frac{\frac{K_2}{\sqrt{k_2}} - \frac{K_1}{\sqrt{k_1}}}{\frac{K_2}{\sqrt{k_2}} + \frac{K_1}{\sqrt{k_1}}} \qquad \dots\dots\dots(11)$$

Since $\gamma e^{-2q_1 a}$ is less than unity equation (11) may be rewritten

$$\bar{\Theta}_i = \frac{2I}{\frac{K_2}{\sqrt{k_2}} + \frac{K_1}{\sqrt{k_1}}} \frac{\sum_{n=0}^{\infty} (-\gamma)^n e^{-(2n+1)\frac{a}{\sqrt{k_1}}\sqrt{p}}}{p\sqrt{p}} \qquad \dots\dots\dots(12)$$

The inverse of this type of expression is known [1] and the interface temperature rise is given by

$$\Theta_c = \frac{4 I \sqrt{c}}{\frac{K_1}{\sqrt{R_2}} + \frac{K_1}{\sqrt{R_1}}} \sum_{n=0}^{\infty} (-\gamma)^n \left[\frac{1}{\sqrt{\pi}} e^{-\left[\beta(n+1)\beta\right]^2} - \left[(n+1)\beta\right]_2 erfc\left[(n+1)\beta\right] \right] \quad \ldots\ldots(13)$$

where $\beta = \frac{a}{\sqrt{k_1 c}}$

3. Discussion

Equation (13) giving the interface temperature rise may be divided into two parts, a summation term which is a function of $\beta = \frac{a}{\sqrt{k_1 c}}$ and

$$\gamma = \frac{\frac{K_2}{\sqrt{R_2}} - \frac{K_1}{\sqrt{R_1}}}{\frac{K_2}{\sqrt{R_2}} + \frac{K_1}{\sqrt{R_1}}}$$

and another factor dependent on time and the thermal properties of the two media. It will be convenient to evaluate the summation term first and it will be noted that $\gamma = \frac{K_2}{\sqrt{R_2}} - \frac{K_1}{\sqrt{R_1}} \bigg/ \sqrt{\frac{K_2}{\sqrt{R_2}} + \frac{K_1}{\sqrt{R_1}}}$ may have

values in the range ± 1. The curves for the summation term are shown in Fig. 2. For values of $\beta > 0.6$ the summation is independent of γ since the terms $n > 0$ are negligible. Under this condition the summation is only a function of β.

The temperature rise for various values of exposure time was then computed for a combustible material having the physical properties of wood, with a protective layer of thickness 5×10^{-2} cms. The temperature rise was specified per unit intensity of incident radiation since these are proportional and the range of thermal constants for the protective layer covered the range of non-metals.

The thermal constants in equation (13) appear as the conductivity and diffusivity of the materials. The temperature rise was therefore computed for various values of conductivity K_1, and thermal capacity per unit volume ρS_1 for the protective layer. From these results a measure of the effectiveness of the protective layer was determined by the time taken for the interface temperature to rise $450^{\circ}C$, a value at which ignition might occur. Fig. 3 shows this protection time plotted as a function of the thermal conductivity of the layer for a range of values of ρS_1. The corresponding results for a range of values of conductivity are shown in Fig. 4.

Figs. 3 and 4 show that for the range of thermal constants considered, the thermal capacity per unit volume of the layer largely controls the protection time though the effect of increasing the thermal conductivity of the layer is to reduce the protection time.

The protection time will of course be dependent on the thickness of the layer a. Fig. 3 shows the protection time plotted as function of the layer thickness, the layer having a thermal conductivity of 1×10^{-4} cal/cm/$^{\circ}$C/sec. and thermal capacity per unit volume of 0.3 cal/$^{\circ}$C/CC. The slope of the graph shows that in the range of thicknesses considered the advantage gained by the addition of further coats increases slightly as the thickness of the layer increases.

Since the protection time depends on the thermal properties of the protected material some materials will require less protection than others. In the following analysis it has been assumed that the thermal properties and thickness of the protective layer remain constant while the properties of the protected material are varied.

Equation (13) shows that under these conditions the temperature rise is governed by the values of the multiplying factor

$$\frac{4\,I\,\sqrt{t}}{\dfrac{K_2}{\sqrt{k_2}}+\dfrac{K_1}{\sqrt{k_1}}} \qquad \text{and} \qquad \gamma = \frac{K_2}{\sqrt{k_2}}-\frac{K_1}{\sqrt{k_1}}\Big/\frac{K_2}{\sqrt{k_2}}+\frac{K_1}{\sqrt{k_1}}$$

These two terms may be rewritten as

$$\frac{4\,I\,\sqrt{t}}{\sqrt{K_2\rho_2 S_2}+\sqrt{K_1\rho_1 S_1}} \qquad \text{and} \qquad \frac{\sqrt{K_2\rho_2 S_2}-\sqrt{K_1\rho_1 S_1}}{\sqrt{K_2\rho_2 S_2}+\sqrt{K_1\rho_1 S_1}} \qquad \text{respectively.}$$

Consider first the effect of increasing the value of thermal conductivity K_2. This will decrease the value of the multiplying factor and increase the value of γ which Fig. 2 shows corresponds to a decrease in the value of the series term in Equation (13). The overall effect therefore is to reduce the temperature rise for a given time of irradiation. Since the thermal properties appear in the equation as the product $K_2\rho_2 S_2$ only, then any variation in the value of $\rho_2 S_2$ will result in the same change in the value of interface temperature rise as a proportional change in the thermal conductivity.

Fig. 6 shows the protection time plotted as a function of the thermal conductivity of the protected material for a range of values of thermal capacity per unit volume for a protective layer of thickness 5×10^{-2}cm, thermal conductivity 2.5×10^{-4}cal/cm/$^{\circ}$C/sec. thermal capacity per unit volume 0.1 cal/$^{\circ}$C/CC. It can be seen from the graph that the materials requiring least protection are those with a high thermal conductivity and high thermal capacity per unit volume.

For convenience the effect of the variables in Equation (13) on the protection time is summarised in Table 1 below.

TABLE 1

The effect of the physical properties of a material and its protective layer on the protection time

Variable		Effect on Protection Time
K_1	Low	Increase
	High	Decrease
$\rho_1 S_1$	Low	Decrease
	High	Increase
a	Thin	Decrease
	Thick	Increase
K_2	Low	Decrease
	High	Increase
$\rho_2 S_2$	Low	Decrease
	High	Increase

4. The effect of heat loss

In the foregoing arguments heat loss from the irradiated surface has been neglected. For a semi-infinite solid receiving radiation the temperature rise of the surface is independent of the heat loss provided

$$\psi \sqrt{\frac{t}{K\rho S}} < 0.12 \qquad (2)$$

where ψ is the heat lost by unit area of the surface in unit time for each degree rise in temperature. A similar condition probably applies to a composite solid. A mean value of ψ for a temperature rise in the range $450°C-550°C$ is $1.4 \times 10^{-3} cal/cm^2/sec/°C$. Hence the time of irradiation for which Equation (13) is valid is given by

$$0.12 = 1.4 \times 10^{-3} \sqrt{\frac{t}{K_1 \rho_1 S_1}}$$

$$\text{or} \quad t = 73 K_1 \rho_1 S_1 \times 10^{-2}$$

Table 2 shows the maximum time of irradiation for which heat loss can be neglected for a range of values of

TABLE 2

The maximum time of irradiation
for which heat loss can be neglected.

$K_1 \rho_1 S_1$	Maximum Time (sec)
1×10^{-6}	.0073
5×10^{-6}	.036
1×10^{-5}	.073
5×10^{-5}	.36
1×10^{-4}	.73
5×10^{-4}	3.6
1×10^{-3}	7.3
5×10^{-3}	36

The results in Table 2 show that in general Equation (13) will only give a correct value for the interface temperature rise for high intensities of irradiation corresponding to short exposure times.
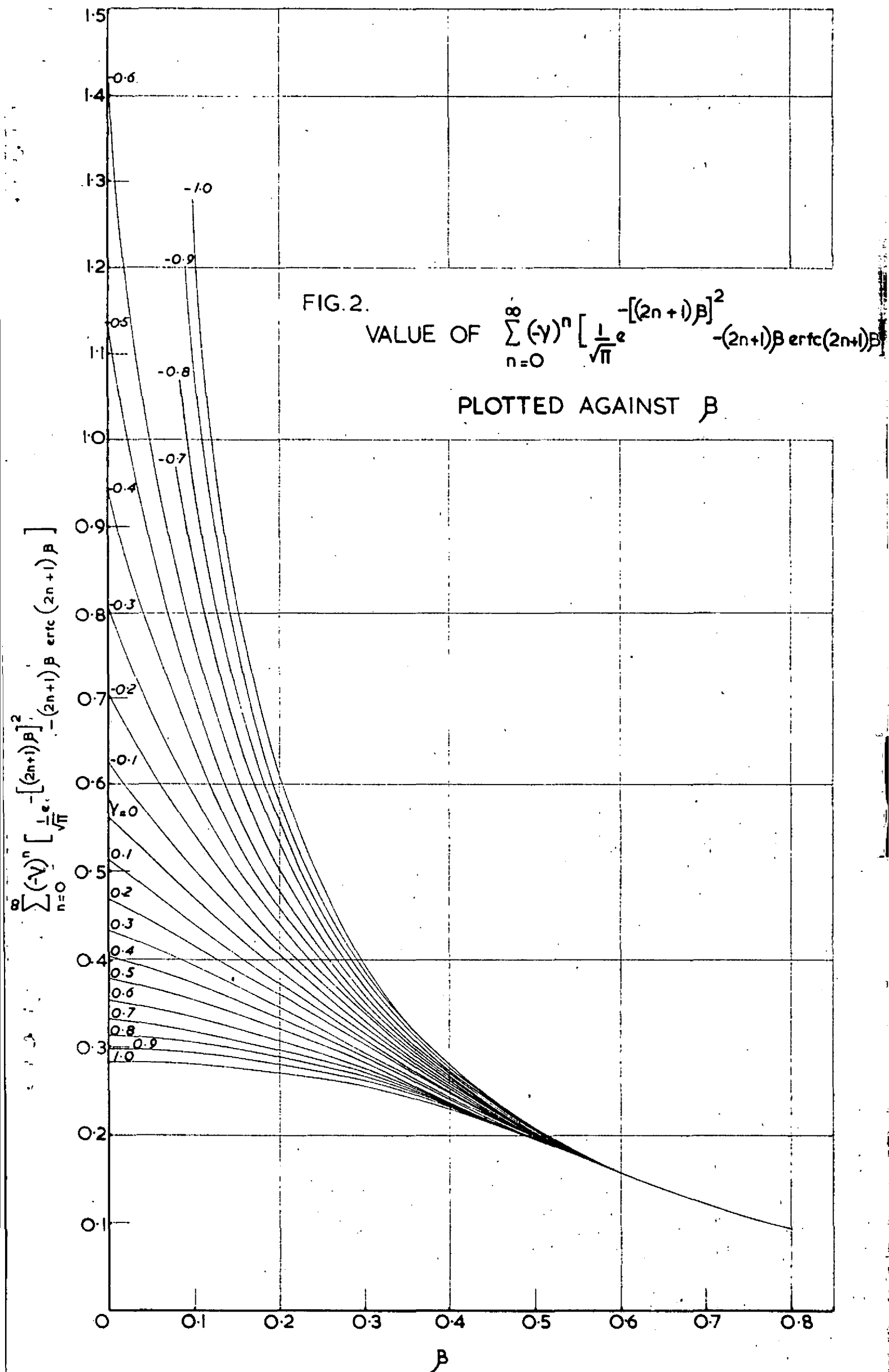
5. Conclusions

An expression has been derived for the temperature rise at the interface of a composite solid with radiation at the surface. Analysis of the expression shows that the most effective fire retardant paint is one with a high thermal capacity and low thermal conductivity, the former condition being the more important. The protection afforded by the paint increases with the thickness of the layer. Materials which require the least protection are those with a high thermal capacity per unit volume and high thermal conductivity.

The validity of the results is limited to short periods of irradiation since heat loss from the irradiated surface was neglected. However the protection times computed from the expression will be shorter than would be obtained in practice and the expression may be used as a guide to the physical properties required of a fire retardant paint.

References

(1)   Dictionary of Laplace Transforms, Part 3A, S.R.E. Dept.
Admiralty, Nov. 1945.

(2)   Lawson, D.I. Fox, L.L. and Webster, C.T.   The Heating of
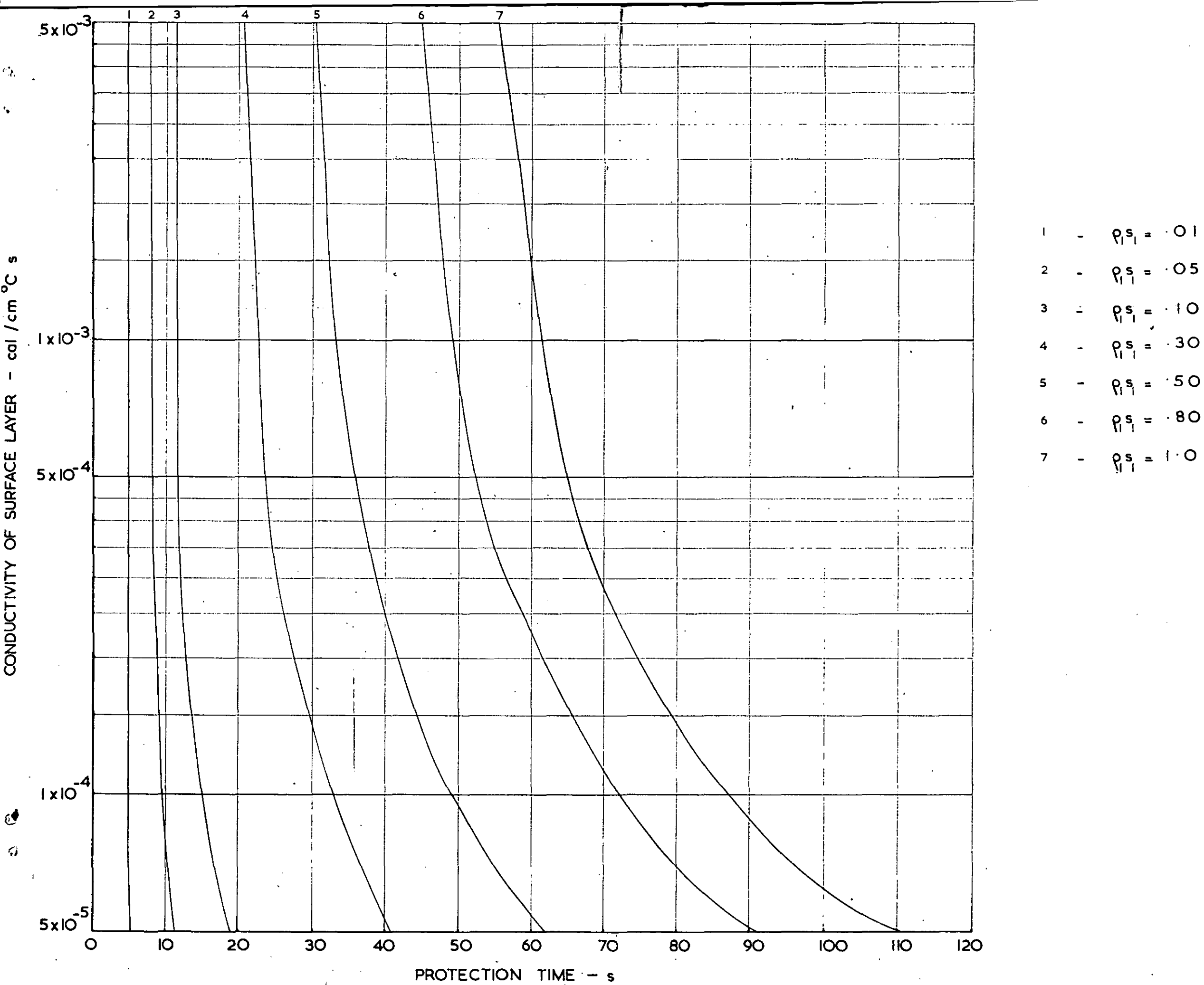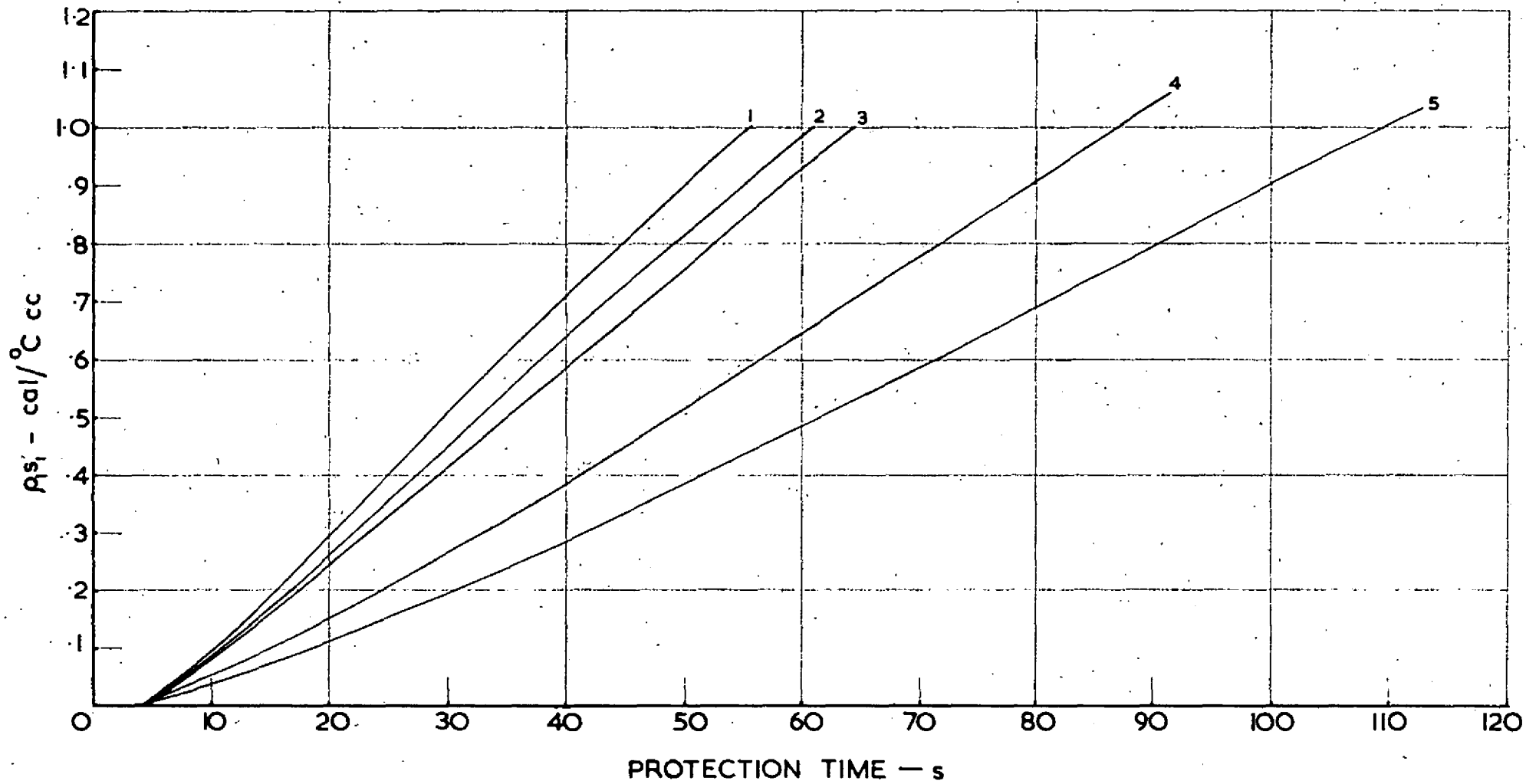Panels by Flue Pipes.   F.P.E. Note No. 32/1950.

ERH

FIG. 2.

VALUE OF $\sum\limits_{n=0}^{\infty} (-\gamma)^n \left[\frac{1}{\sqrt{\pi}} e^{-[(2n+1)\beta]^2} - (2n+1)\beta \, \text{ertc}(2n+1)\beta \right]$

PLOTTED AGAINST β

FIG. 3.   VARIATION IN PROTECTION TIME WITH THERMAL CONDUCTIVITY.

FIG. 4.   VARIATION   IN PROTECTION   TIME   WITH   THERMAL   CAPACITY   PER   UNIT   VOLUME.
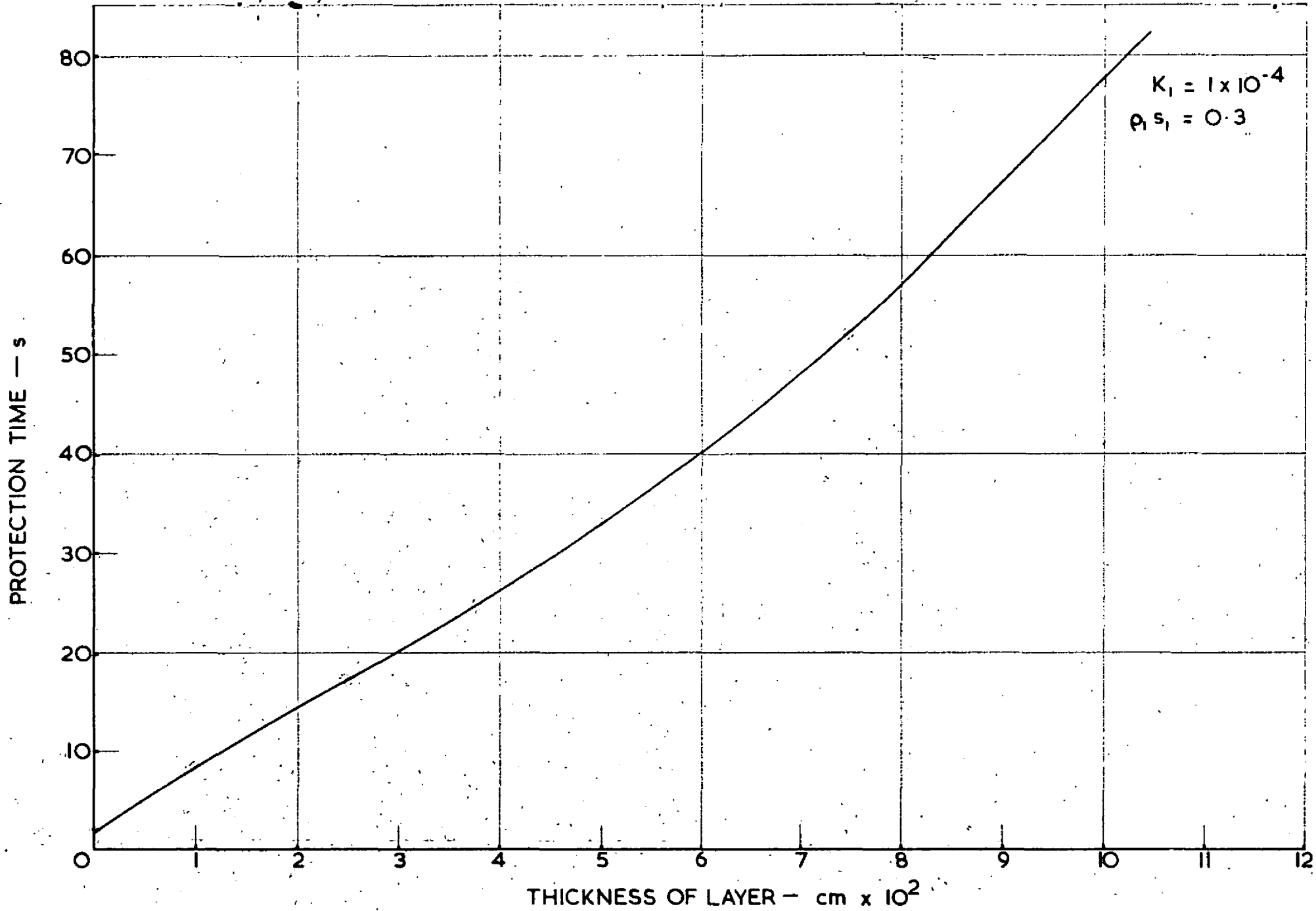
FIG. 5.   VARIATION IN PROTECTION TIME WITH THICKNESS.
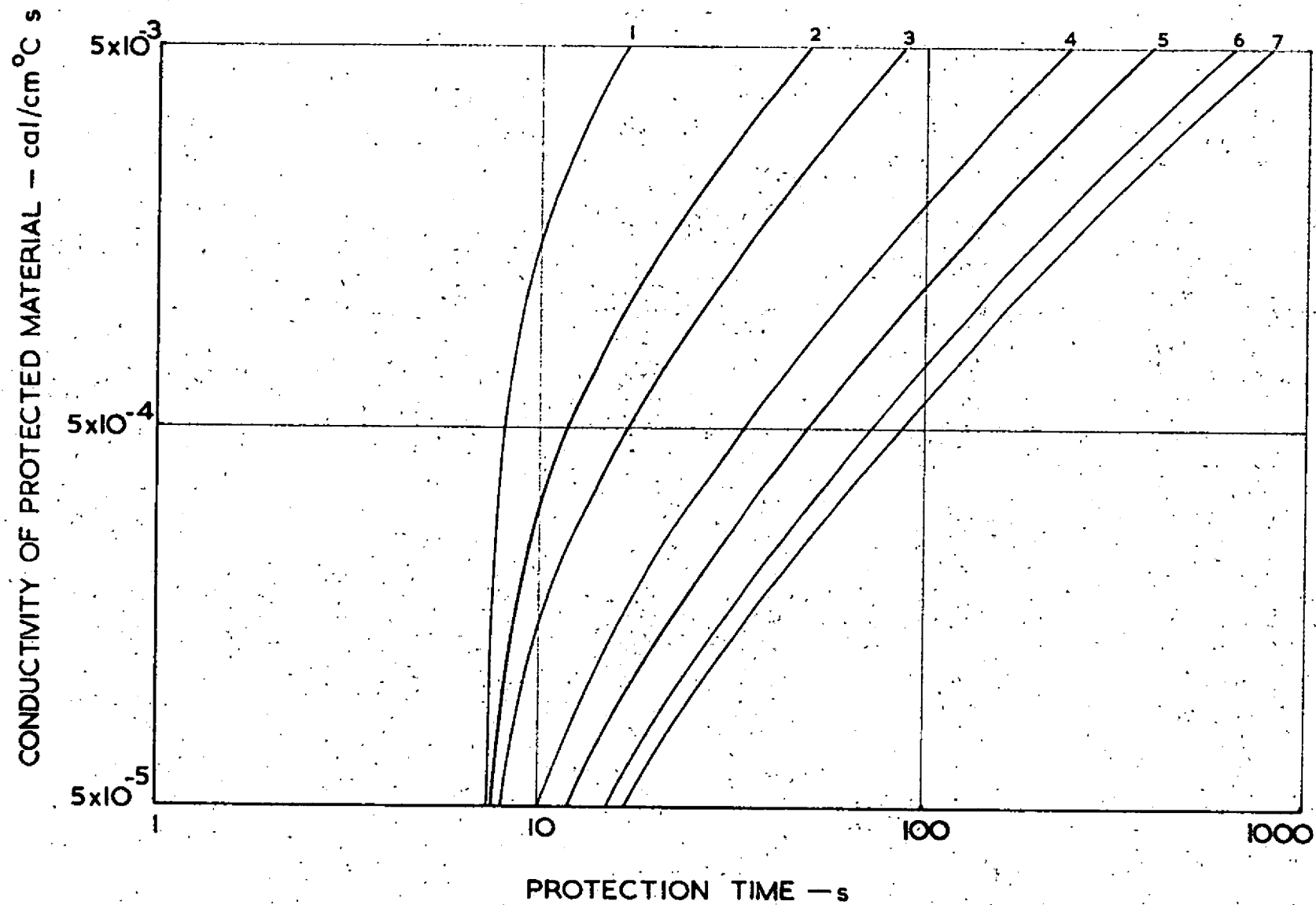
FIG. 6. VARIATION IN PROTECTION TIME WITH THERMAL CONDUCTIVITY OF PROTECTED MATERIAL.