Practical Estimation of Deflection and Ultimate Strength of Fire Exposed Composite Slab

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ABSTRACT

The behavior of the fire exposure ¹⁾. The difficulty of the step by step calculus from the fire start arises in this time zone. But the required estimation is for the 1 or 2 hours after fire start, then the behavior of the steel deck composite floor slab becomes stable, and it becomes possible to calculate the stress and deflection with the aid of the section properties of the effective section (as shown in the references 1) to 5), for the up- and downward bending and by setting up the stress deflection formula considered up- and downward bending zone. In this paper, the author developes a practical calculation method for the estimation of deflection and ultimate strength of the fire exposed steel deck composite floor slab by this means, and verifies the validity of this method by comparing to experimental results.

KEY WORDS: Steel Deck Composite Floor Slab, Fire Exposed, Deflection, Ultimate Strength, Estimation Calculus

NOTATIONS

F(x,y); temperature

Ec, Es ; elastic modulas of concrete(c), steel(s)
Fc, Fs ; strength of concrete(c), steel(s)

 $K_{+}K_{-}K_{M}$; stiffness of downward($_{+}$), upward($_{-}$), end_restraint($_{M}$)

 M_{+}, M_{-} ; bending moment downward($_{+}$),upward($_{-}$)

y ; deflection

Dadd, Wadd; additional deflection and load by the end restraint

INTRODUCTION

The steel deck composite floor slab without fire protection is a popular and standard floor construction method for steel, reinforced concrete, and mixed structures. In Japan, since 1987 this slab has been used. GOSEI SLAB KOGYO KAI (Steel deck composite floor institute) was founded this year and has performed large amounts of heating tests of loaded composite floor slabs. Until December 1998 we finished the 104 th tests for the approval. Based on the results of these tests, we started a study five years ago for setting up a caluculation method for the estimation of deflection under service load and ultimate strength for the 0.5 to 2 hour fire exposed composite floor slab with a steel deck. This study was proceeded with Prof.Uesughi, Chiba University. We added basical pure bend loading tests for this study.

The purpose of this paper is to introduce the result of our five years study. Some new ideas are included but the substance of the procedure is based on the elementary beam theory as shown in our classical text book⁵⁾ and recent Euro code³⁾. The behavior of the fire exposed composite floor slab is shown in the second chapter, the possibility of the setting up of the section properties is indicated there. The procedure for the setting up of the section properties, and the span - deflection relations and the ultimate load table are described in the next two chapters. In these two chapters, it is pointed out that the consideration of the up- and downward bending zone is the key of good estimation results. The 104 experimental results are compared with this estimation method.

Through this paper, it is shown that the design caluculation of the fire exposed composite slab becomes the same as the usual structural design caluculation by using table of section properties and stress deflection formulas.

BEHAVIOR OF FIRE EXPOSED COMPOSITE FLOOR

The behavior of the loaded (as well as unloaded since self weight is imposed) composite floor slab under the fire test is very familiar to the fire experimenter, in Europe, Asia,USA, Japan, etc. FIGURE 1 shows the relation of the deflection, the thermal curvature restraining, and the stiffness to the fire exposed hours. The first 15 to 20 minutes is very eventful, during these minutes, the steel deck is detached from the concrete portion by thermal buckling, the stiffness reduces. The deflection after this depends on the curvature restraint and the reduction of the stiffness of the section. For economy and to reduce building weight, the imposed load is as large as permissible and the deflection-heating time relation will most often follow curve (2) in FIGURE 1. But sometimes the relation will follow curve (1), this occurs in cases b, and c, as shown in FIGURE 2. When the composite floor is not fixed on the supporting beam, the deflection increases with time due to thermal curvature, i.e. the Bi-metal effect, this is case a in FIGURE 2.

After 15 to 20 minutes, the steel deck will be detached from the concrete portion 1),3), and the concrete portion will be smered with cracks (the distance between the crack is 5 to 10 cm when observed after fire test or after the time of the evaporation of moisture) in the tension zone, bending is resisted by the reinforcing and the concrete of the compression zone. By fixing of the ends of the composite slab on the supporting beam, the restraint of the thermal curvature will raise the upward bending moment through the span. The behavior of the restraint of the thermal

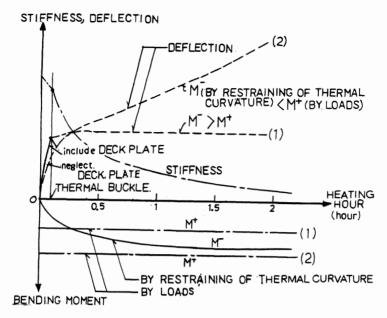


FIGURE 1 Behavior of fire exposed steel deck composite floor

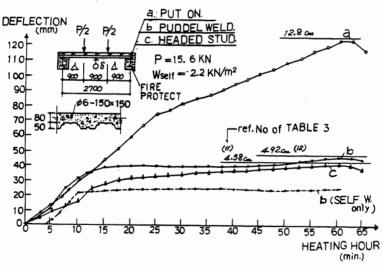


FIGURE 2 A sample of fire test

elongation is very complicated, because the the zone of the fire protected support has the composite action of the steel deck and the concrete i.e. incomplete composite action, and the thermal elongation and the contraction by deflection is raised at the same time. These actions are of the same order of magnitude.

Anyway 15 to 20 eventful minutes later, the structural model of the composite slab becomes clear, along the span the section is smeared by cracks according to the up- and downward bending, the steel deck is already detatched from the concrete portion. Our required FIRE_RATE is usually 0.5 to 2 hours. The design calculation treates this stable state of the fire exposed composite floor slab. The deck plate can be neglected from the calculation as shown in the Eurocode 4, part 1.2,4.3.1.3.(6) ³).

CALCULATION OF THE SECTION PROPERTIES

Since the thermal state of the section is known, the calculations can be made. To get the thermal data of the cross section, either direct measurements or available calculation methods are used. The numerical integration for searching of the neutral axis becomes easy if the thermal state is represented a continuous function. Here, we use the 3 degree algebraic function along the depth of the section with the aide of 8 thermal data (4 at the valley, 4 at the crown), and the trigonometric function along the width of the section. The trigonometric function can be used to represent the difference of the temperature at the crown and the valley. The detail of these functions is shown in TABLE 1. FIGURE 3 is an example of the function fitting.

The material constants are a function of temperature, a large amounts of the function was proposed⁵). Here, because of the the recent proposal, we use the Eurocode No 4³). For the same reason as the above, we represent these by continuous functions as shown in FIGURE 4, in this figure the three graphs correspond to the temperature of the valley, the crown section and the top surface.

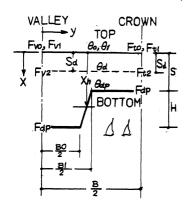
The section properties of the effective section (we neglect the tension stress of the concrete) for the up- and downward bending is calculated by the numerical integration, a fine mesh is required along the depth. Here, we use Romberg integral along the depth and the cell summation along the width. TABLE 2 is a sample of the chart of the section properties. The thermal curvature and the up- and downward ultimate bending moment are included in this chart, i.e. calculated as the section properties.

SPAN-DEFLECTION, ULTIMATE LOAD

After the section properties are calculated, we can calculate stresses and the deflection of the composite floor slab by the elementary beam theory with the account of the end restraint of the rotation and the up- and downward bending zone. This is important and essential, because the stiffness is fairly different in both zones.

The spring constant of the end restraint was found out from the result of heating tests imposed pure bending load. The method of the pure bend loading, the result of experiments and its evaluation are shown in FIGURE 5. Here we use the secant of the load deflection relations.

TABLE 1 Thermal functions



(A) THERMAL DATA

SYMBOL	MEAN TO	UNIT
Fuo, Fuo	TEMPERATURE OF TOP. TEMP GRADIENT OF TOP.	°C
Fvi, Fti	TEMP GRADIENT of TOP.	°C/cm
Fv2, Ft2	TEMP, at X=Sa.	°C
Fdp	TEMP at X=Sa. TEMP of BOTM = in FURNACE	c

(D) DIMENSIONS

	SYMBOL	MEAN TO	UNIT
	Xh	DISTANCE from TOP to BOTTOM	
	Sa	X=Sa	
į	BO	WIDTH of UNER VALLEY	
	Bit	✓ UPPER VALLEY	cm
		PITCH of GROOVE	
	H	HIGHT OF STEEL DECK PL	
	S	DEPTH of SLAB	

(C) TEMPERATURE ALONG THE HIGHT OF VALLEY AND CROWN

SYMBOL			UNIT
n	$\frac{l_n\left(\frac{BO+B^1}{2\cdot B}\right)}{l_n\left(\frac{BO+B^1}{2\cdot B}\right)}$		_
(A)	$\frac{F_{v1}-F_{t1}}{2}\cdot\cos\left\{180\cdot\left(\frac{2y}{B}\right)^{n}\right\}+\frac{F_{v1}+F_{t1}}{2}$	X=0 TEMP	1
θ 。 (Υ)	$\frac{Fw - Fto}{2} \cdot cos \left\{ (2y)^n \right\} + \frac{Fv + Fto}{2}$	X=0 TEMP.	°C/cm
θ a (λ)	$\frac{F_{V2}-F_{t2}}{2}\cdot\cos\left\{180\cdot\left(\frac{2y}{B}\right)^n\right\}+\frac{F_{V2}+F_{t2}}{2}$	X=Sa TEMP.	oc.
Өфр	Fdp	TEMP AT BOTTOM	

(d) TEMPERATUR AT (X, Y)

SYMBOL	FORMULA	
а	$\frac{1}{S_d - X_h} \left\{ \frac{(\theta d - \theta_f \cdot Sd - \theta_o)}{Sd^2} - \frac{(\theta dp - \theta_f \cdot X_h - \theta_o)}{X_h^2} \right\}$	°C/cm³
۵	$\frac{1}{S_{\mathbf{d}} - \lambda_{\mathbf{h}}} \left\{ -\frac{\lambda_{\mathbf{h}} (G_{\mathbf{d}} - \theta_{\mathbf{h}} \cdot S_{\mathbf{d}} - \theta_{\mathbf{h}})}{S_{\mathbf{d}}^{2}} + \frac{S_{\mathbf{d}} (\theta_{\mathbf{d}p} - \theta_{\mathbf{h}} \lambda_{\mathbf{h}} - \theta_{\mathbf{h}})}{\lambda_{\mathbf{h}}^{2}} \right\}$	°C/cm²
С	θ_{ℓ}	°C/cm
d	θ•	°C
F(x, y)	$a X^3 + b X^2 + C X + d$ TEMP AT X,Y	2

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F=F (Xy): TEMPERATATURE of SECTION at (X,Y)  
E\infty; ELASTIC MODULAS of CONCRETE at 20 °C  
E\infty; STEEL  
F\infty; UPPER YIELD STRESS of CONCRETE at 20 °C  
F\infty; STEEL  
\alpha_{C} = \alpha_{S} = 1 \times 10^{-5}; THERMAL ELONGATION FACTOR of CONCRETE and STEEL  
Ec Fc; ELASTIC MODULAS, UPPER YIELD STRESS of CONCRETE at F\inftyC  
Es Fs; STEEL  
Ec(F)/E\infty = \frac{1}{2}[exp[-3.844×10<sup>-3</sup>(F-200)-4.66×10<sup>-2</sup>]]  
Fc(F)/Fs\infty = \{0.5 \cos{\left(\frac{\pi \cdot F}{1200}\right)} + 0.5\}^{1.25}  
Es(F)/E\infty = 0.5 \cos{\left(\frac{\pi \cdot F}{1200}\right)} + 0.5\}^{1.65}
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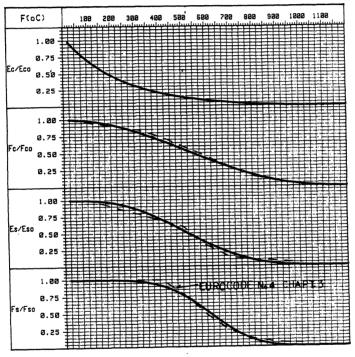


FIGURE 3 Function fitting to the material constants

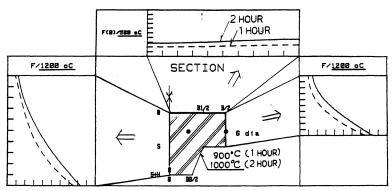


FIGURE 4 Sample of thermal function

TABLE 2 Sample of Sheet of Section Properties

$C50 - S90 - \phi6$; 100 x 100

SECTION PROPERTIES under the 2 H fire_rate.

(1) GROWBAL_SECTION

- (2) EFFECTIVE_SECTION for the downward bending.

 Ka_ep(kgf/m)= 1.7105E+07

 Xn_ep(cm)= 1.4628E+00

 K_ep(kgf*cm2/m)= 4.4892E+07

 1/R_ep(1/cm)= 1.5961E-03

 E_ep(1/cm)= 3.1210E-03
- (3) EFFECTIVE_SECTION for the upward bending.

 Ka_en(kgf/m)= 1.5150E+07

 Xn_en(cm)= 7.1149E+00

 K_en(kgf*cm2/m)= 4.8778E+07

 1/R_en(1/cm)= 3.8653E-03

 E_en(kgf*cm/m)= 4.8860E-03
- (5) ULTIMATE_MOMENT for the upward bending.

 Xn_un(cm)= 9.4384E+888

 M_un(kgf*cm/m)= 1.1313E+85

The spring constant and the ultimate bending moment is found out from this experiment., the results is as follows.

HEADED STUD 16 mm dia.: KM=6.65x10³ KN cm/rad, Mu=200 KN cm

PUDDLE WELD 25 mm dia.: KM=2.98x10³ KN cm/rad, Mu=125 KN cm

The ultimate bending moment of the puddle weld 25 mm dia. is a half of the headed stud 16 mm dia.

The procedure of the calculation of the stress, the deflection, and the ultimate load are shown in FIGURE 6.

First we calculate the maximum up- and downward bending moments and its zone under the uniformly distributed loads, for an example 10 KN/m².

The ultimate load is calculated using the ultimate up- and downward bending moments, these are calculated as one of the section properties.

Next we impose the corresponding thermal curvature, calculated already as the section properties, to the up- and downward bending zone calculated above, and find out the upward bending moment and the deflection at the mid_span. Also we calculate the equidistributed uniformly loads which raises the downward bending moment, the value equal to the above upward bending moment at the mid-span, we call this deflection Dadd and the corresponding load Wadd. Finally add the Dadd to the above deflection and the Wadd to the above ultimate load.

These values represent the estimation of the deflection and the ultimate load.

The calculation of the up- and downward bending zone may also be done under the imposing of the vertical loads and the thermal curvature at same time, but the crack of the tension zone of the concrete may be raised before the restraint of the thermal curvature develops. Here, we find the up- and downward bending zone by imposing the vertical loads only.

The formula of the above calculation is not so familiar, this formula is shown in APPENDIX for convenience.

The neglecting of the upward bending due to the restraint of the thermal curvature³) gives very conservative estimation for the ultimate load and the deflection. Since 1985, the specimen of our regulated experiment for the approval of the loading capacity and its corresponding fire rate of the steel deck composite floor slab has the end restraint that represents the real frame structure. The size of the supporting beam used in the experiment is selected as the same tortional stiffness of the real frame structure, i.e. H-400mm x 250mm for 6m span to H-600mm x 200 mm for 7

to 8 m span under the 400 °C. Our classical textbook⁵⁾ shows clearly the importance of the end restraint and its merit for the fire resistance of the slab and the beam.

From the start of the experiment, the evaluation of of the upward bending, brought on by the end restraint, is the annoying problem, rough integration brings capricious results, sometimes correct somtimes inaccurate. But we still propose that this method be useful since it is a very effective method from a practical point of view.

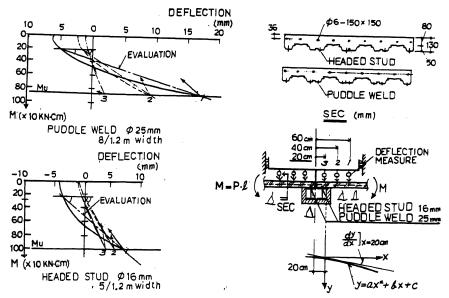


FIGURE 5 Modeling of end restraint

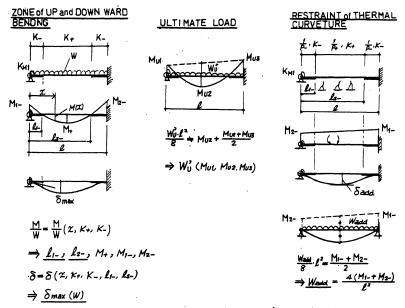
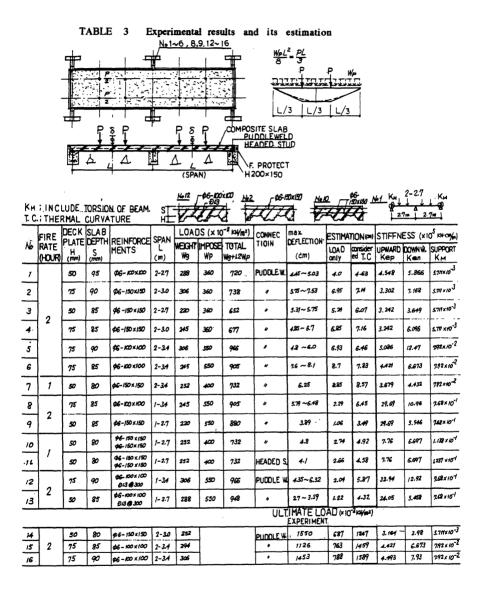


FIGURE 6 Procedure of deflection ultimate load calculus

COMPARISON WITH RESULTS OF EXPERIMENTS

TABLE 3 shows the comparison of the experimental results and the estimation. The torsional stiffness of the supporting beam is considered for the spring constant of the end. The deflection and the ultimate load is estimated fairly well. The 104 experiments are included in this table. The term of the STIFFNESS shows the upward, downward and end restraint stiffness.



CONCLUDING REMARKS

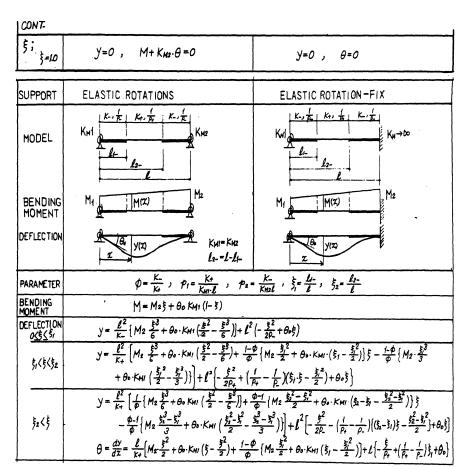
A practical procedure for the estimation of the deflection and the ultimate load of a fire exposed composite floor slab is developed using an elementary method, and this procedure can fairly well estimate the results from experiments. The consideration of the restraint of the thermal curvature and the up- and downward bending zone is an important key in this procedure.

The deflection of the floor slab under the fire should be restricted to allow safe fire fighting. So fire protection of the beams is necessary and more protection is required for columns.

The profile of the steel deck plate treated in this paper is only a trapenzoidal one, and the service load for the calculation of the deflection is 1/3 to 1/2 of the ultimate load. This reflects what is common on the the commecial market in Japan.

APPENDIX: Formula of bending moments and deflection

SUPPORT	ELASTIC ROTATIONS	ELASTIC ROTATION—FIX
MODEL BENDING MOMENT DEFLECTION	K_{H} K	M(x) M ₊ M(x) M ₊ M(x) M ₊
PARAMETER	$\dot{\xi} = \frac{z}{l}$; $p = \frac{K_{+}}{K_{H}l}$, $\phi =$	$\frac{K_{-}}{K_{+}}, \dot{\xi}_{1} = \frac{\ell_{-}}{\ell}, \dot{\xi}_{2} = \frac{\ell_{2-}}{\ell}$
BENDING MOMENT	$M = -\frac{w\ell^2}{2}(\xi - \xi_i)(\xi - l + \xi_i)$	$M = -\frac{\mathcal{W}\ell^2}{2}(\xi - \xi_1)(\xi - \xi_2)$
DEFLECTION- 0<\$<\$,	$y = \frac{w t^4}{24 k_t^4} \left[\frac{1}{\phi} \left\{ \frac{\xi^4}{12} - \frac{\xi^3}{6} + \frac{\xi_1(t-\xi_1)}{2} \xi^2 \right\} + p \cdot \xi_1(t-\xi_1) \xi \right]$	$y = \frac{w \mathcal{E}^4}{2 \kappa_1} \left\{ \frac{1}{\Phi} \left(\frac{\xi^4}{72} - \frac{\xi_1 + \xi_2}{6} \xi^3 + \frac{\xi_1 + \xi_2}{2} \xi^2 \right) + \mathcal{P} \xi_1 \xi_2 \cdot \xi \right\}$
\$1<\$< \$2	$y = \frac{w \cancel{\xi}^4}{2 \cancel{k}^4} \left[\frac{\cancel{\xi}^4}{12} - \frac{\cancel{\xi}^3}{6} + \frac{\cancel{\xi}_1(1-\cancel{\xi}_1)}{2} \cancel{\xi}^2 + \frac{1-\phi}{\phi} \left\{ \frac{\cancel{\xi}^3}{3} - \frac{\cancel{\xi}^2}{3} - \frac{\cancel{\xi}^2}{2} + \frac{1}{\cancel{\xi}^3} \frac{\cancel{\xi}^3}{2} + \frac{1}{\cancel{\xi}^3} \cancel{\xi$	$J = \frac{W\ell^4}{2Kr} \left\{ \frac{3^4}{12^2} - \frac{\xi_1 + \xi_2}{6} \xi^3 + \frac{\xi_1 + \xi_2}{2} \xi^3 + \frac{1-\phi}{\phi} \left(\frac{3^3}{3} - \frac{\xi_1 + \xi_2}{2} \xi_1^2 + \frac{\xi_2}{7} \xi_2 \right) \right\}$ $+ \frac{1-\phi}{4} \left\{ - \left[-\frac{\xi_1^4}{4} + \frac{\xi_1 + \xi_2}{3} \xi_1^3 - \frac{\xi_1 + \xi_2}{7} \xi_1^3 + P + \xi_1 \xi_2 \right] \right\}$
\$2 \ \$ \ 1	$\frac{\tau_{\mathcal{H}} \vee \tau_{\mathcal{H}}}{\phi} = \frac{\tau_{\mathcal{H}} \cdot \tau_{\mathcal{H}}}{\phi} = \frac{\tau_{\mathcal{H}}}{\phi} = \frac{\tau_{\mathcal{H}}}{\phi}$	$y = \frac{w \beta^{4}}{2kr} \left\{ \frac{f}{\phi} \left(\frac{\xi^{4}}{I^{2}} - \frac{\xi_{1}r + \xi_{2}}{G} \xi^{3} + \frac{\xi_{1}' + \xi_{2}}{2} \xi^{2} \right) + \beta^{-1} \xi_{1} \xi_{2} \cdot \xi^{2} \right\} $ $+ \frac{1 - \phi}{\phi} \left\{ \frac{\xi_{3}^{3} - \xi_{3}^{3}}{3} - \frac{\xi_{1}r \xi_{2}}{2} (\xi_{2}^{2} - \xi_{1}^{2}) + \xi_{1} \xi_{2} (\xi_{2} - \xi_{1}) \right\} \cdot \xi$ $+ \frac{1 - \phi}{\phi} \left\{ \frac{\xi_{3}^{2} - \xi_{1}^{4}}{3} - \frac{\xi_{1}r \xi_{2}}{2} (\xi_{2}^{2} - \xi_{1}^{2}) + \frac{\xi_{1} \xi_{2}}{2} (\xi_{2}^{2} - \xi_{1}^{2}) \right\} \cdot \xi$ $+ \frac{1 - \phi}{\phi} \left\{ \frac{\xi_{3}^{2} - \xi_{1}^{4}}{4} - \frac{\xi_{1}r + \xi_{2}}{3} (\xi_{2}^{2} - \xi_{1}^{2}) + \frac{\xi_{1} \xi_{2}}{2} (\xi_{2}^{2} - \xi_{1}^{2}) \right\}$



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